Transforming a Perceptron to go through origin

- Can be easier to implement/analyze classifiers of the form:

$$h(x;\theta) = \begin{cases} +1 & \text{if } \theta^T x > 0 \\ -1 & \text{otherwise} \end{cases}$$

- Without an explicit offset term θ_0 → must pass through origin
  ↳ appears limiting
  - ↳ We can convert linear separator with offset → no offset but higher dimension

- Consider the d-dimensional linear separator defined by $\theta = [\theta_1 \; \theta_2 \; \cdots \; \theta_d]$ and offset θ_0
  - To each data point $x \in D$, append a coordinate with value $+1$, yielding

$$x_{new} = [x_1 \; \cdots \; x_d \quad +1]^T$$

  - And we define:

$$\theta_{new} = [\theta_1 \; \cdots \; \theta_d \;\; \theta_0]^T$$

  - Then:

$$\theta^T_{new} \cdot x_{new} = \theta_1 x_1 + \ldots + \theta_d x_d + \theta_0 \cdot 1$$
$$= \boxed{\theta^T x + \theta_0}$$

  - Thus, θ_{new} is an equivalent $(d+1)$-dimensional separator but with no offset

- Consider the dataset:

$$X = [[1], [2], [3], [4]]$$
$$Y = [[+1], [+1], [-1], [-1]]$$

  - This is linearly separable in $d=1$ with $\theta = [-1]$ and $\theta_0 = 2.5$. However, not separable through the origin.
  - Now, we let:

$$X_{new} = \left[\begin{bmatrix}1\\1\end{bmatrix} \begin{bmatrix}2\\1\end{bmatrix} \begin{bmatrix}3\\1\end{bmatrix} \begin{bmatrix}4\\1\end{bmatrix} \right]$$

  - This new dataset is separable through the origin with $\theta_{new} = [-1 \quad 2.5]^T$