Quadric Surfaces

- Any surfaces in the general form:

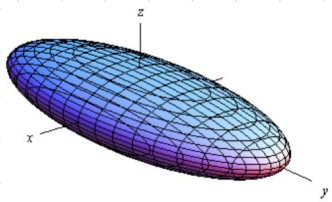
$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

where $A, \ldots, J$ are constants.

- Some common quadric surfaces:

Ellipsoid

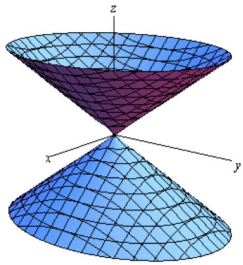
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



Sphere $\longrightarrow$ $a = b = c$

Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



$$z^2 = c^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = A^2x^2 + B^2y^2$$

$$z = \pm\sqrt{A^2x^2 + B^2y^2}$$

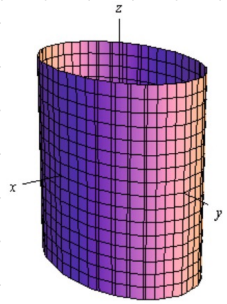
$+ \longrightarrow$ Top cone

$- \longrightarrow$ Bottom cone

Open along x-axis instead: $\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{x^2}{a^2}$

Cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

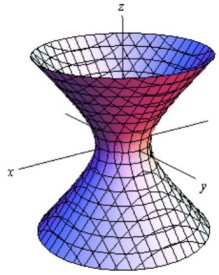


If $a = b$, the cross-section is circular, such that

$$x^2 + y^2 = r^2$$

Hyperboloid of One Sheet

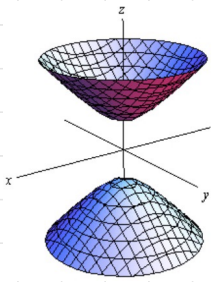
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



- Variable with negative in front of it will give the axis along which the graph is centered

Hyperboloid of One Sheet

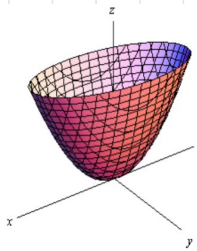
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



- Variable with positive in front of it will give the axis along which the graph is centered
- Opposite of 1-sheet hyperboloid

Elliptic Paraboloid

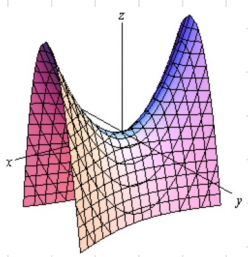
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$



- $a = b \rightarrow$ .S. will be circle
- Variable that isn't squared is the axis upon which the paraboloid opens up
- Sign of c determines the direction that the paraboloid opens
  - ↳ + = up, - = down

Hyperbolic Paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$$



- Saddle-shaped
- Sign of c determines direction of surface that opens up
  - c is positive

Ex, $z = -x^2 - y^2 + 6$

- Elliptic paraboloid that opens downward (- is on x & y instead of z)
- Starts at $z = 6$ instead of $z = 0$