Tabu Search on a Tiny TSP (5 cities): Setup

Cities: A, B, C, D, E (symmetric distances)

	A	B	C	D	E
A	0	4	7	3	6
B	4	0	2	6	3
C	7	2	0	5	4
D	3	6	5	0	2
E	6	3	4	2	0
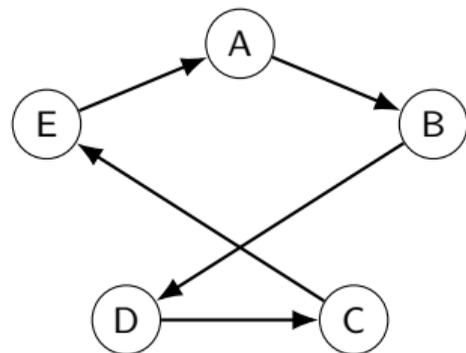
Initial tour s_0
cost 25

State: a tour permutation.
Start from:

$$s_0 = A - B - D - C - E - A, \qquad E(s_0) = 25$$

Move: 2-opt (cut two edges, reconnect, reverse segment).

Tabu Iteration 1: Take Best Move, Record Tabu Attributes

From s_0 we pick a strong 2-opt move: cut (B, D) and (C, E), add (B, C) and (D, E).
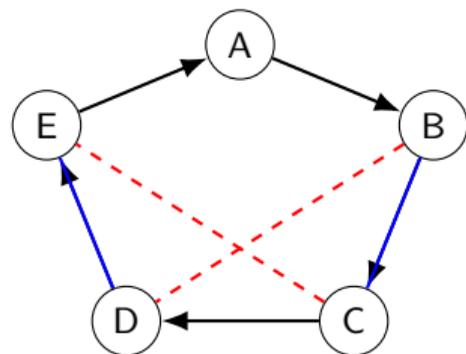
Cost update:

$$E(s_1) = 25 - (6 + 4) + (2 + 2) = 19$$

$$s_1 = A - B - C - D - E - A$$

Tabu update (tenure $L = 2$):
store added edges as tabu attributes:

$$\text{Tabu} \leftarrow \{(B, C), (D, E)\}$$

Interpretation: "don't immediately undo this improvement by removing these edges."

s_1 cost 19
Tabu: $(BC), (DE)$

Tabu Iteration 2: Best Move Might Be Tabu (Prevent Cycling)

At s_1 (cost 19), one tempting move is to **reverse the previous change** (which would *remove* (B, C) and (D, E)).
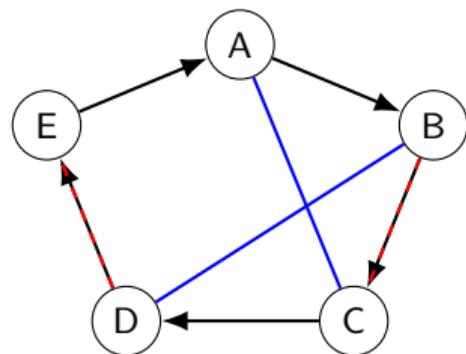
That reverse 2-opt would:

remove $(B, C), (D, E) \quad \Rightarrow \quad$ **tabu (blocked)**

because (B, C) and (D, E) are in the tabu list.

So Tabu Search picks the best admissible neighbor instead.

Example admissible 2-opt: cut (A, B) and (C, D), add (A, C) and (B, D):

$$\Delta E = (7 + 6) - (4 + 5) = +4 \quad \Rightarrow \quad E = 23$$

Tabu prevents undo
Candidate adds $(AC), (BD)$

Aspiration Criterion (Override Tabu If It Beats the Best-So-Far)

Best-so-far after Iteration 1:

$$s^\star = s_1, \qquad E(s^\star) = 19$$

Sometimes a move is **tabu** but leads to a solution better than $s^\star$. In that case we allow it via **aspiration**:

Aspiration rule (simple)

If a tabu move produces $E(s') < E(s^\star)$, **accept it anyway**.

Interpretation: tabu prevents *cycling*, not *progress*. We block short-term reversals unless they give a genuine breakthrough.

> **TS at a glance**
> 1) evaluate neighborhood
> 2) pick best *admissible* move
> 3) update tabu list
> 4) update best-so-far
> 5) aspiration can override tabu

Aspiration in Tabu Search (Worked Example)

Current solution:

$$E(s_k) = 20, \qquad \text{best-so-far } E(s^\star) = 19$$

Tabu list: recent moves $\{m_1, m_2\}$

Candidate neighbors:

Move	Tabu?	Cost
m_1	yes	17
m_3	no	18
m_4	no	21

Aspiration rule:

Override tabu if

$$E(s') < E(s^\star)$$

Decision: m_1 is tabu but produces the best solution so far $\Rightarrow$ accept it.