Example Problem

Minimize the function:

$$f(x, y) = (x - 2)^2 + (y + 1)^2$$

Properties:

- Continuous
- Convex
- Global minimum at $(2, -1)$

Note: This example illustrates GA mechanics, not algorithm choice.

Initialization

Initial population (size $= 4$):

Individual	x	y
1	0.5	-0.2
2	3.8	-2.0
3	1.5	1.0
4	4.2	-0.5

$$f(x, y) = (x - 2)^2 + (y + 1)^2$$

Fitness Evaluation

Evaluate each individual:

ID	x	y	$f(x,y)$
1	0.5	-0.2	2.89
2	3.8	-2.0	4.24
3	1.5	1.0	4.25
4	4.2	-0.5	5.09

$$f(x,y) = (x-2)^2 + (y+1)^2$$

Note

Lower fitness values are preferred (minimization).

Selection

ID	x	y	$f(x, y)$
1	0.5	-0.2	2.89
2	3.8	-2.0	4.24
3	1.5	1.0	4.25
4	4.2	-0.5	5.09

Assume tournament selection (size $= 2$).

Selected parents:

- Individual 1
- Individual 2

Note

Better solutions are favored but not guaranteed.

Crossover Step

Apply arithmetic crossover with $\alpha = 0.4$:

$$\mathbf{x}^{(c)} = 0.4(0.5, -0.2) + 0.6(3.8, -2.0)$$

$$\mathbf{x}^{(c)} = (2.48,\ -1.28)$$

Offspring lies on the line segment between parents.

Mutation Step

Apply Gaussian mutation with $\sigma = 0.1$:

$$\Delta = (-0.03,\ 0.05) \quad \text{(one Gaussian draw)}$$

$$\mathbf{x}' = (2.45,\ -1.23)$$

Note

Small perturbations refine solutions locally.

New Population Member

New offspring:

$$\mathbf{x}' = (2.45,\ -1.23)$$

Fitness:

$$f(2.45, -1.23) = 0.25$$

Note

GA operators rapidly move solutions toward the optimum.

One-Generation Summary

In one generation:

- Poor solutions are discarded
- Better offspring are introduced
- Population shifts toward the optimum

Note

Evolution is incremental but directional.

Why Real-Valued GA Works Well Here

- Search space is continuous
- Operators preserve geometry
- Mutation enables fine local refinement

Note

Real-valued GA aligns naturally with the problem structure.