Worked Example: Permutation GA on a 7-City TSP

Goal: minimize tour length (return to start)

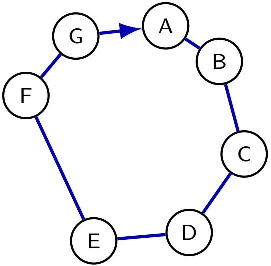
Cities: $\{A, B, C, D, E, F, G\}$
Chromosome (permutation):

$$\pi = [\pi_1, \pi_2, \ldots, \pi_7]$$

$$L(\pi) = \sum_{k=1}^{6} d(\pi_k, \pi_{k+1}) + d(\pi_7, \pi_1)$$

We will demonstrate one generation:

- Evaluate fitness (tour length)
- Tournament selection
- PMX crossover (step-by-step)
- Swap mutation
- Replacement with elitism



tour = permutation + return

Note

Permutation GA requires permutation-aware crossover (PMX).

Distance Data (Example Instance)

Symmetric distance matrix
(units arbitrary)

	A	B	C	D	E	F	G
A	0	4	3	7	3	6	8
B	4	0	4	6	7	3	5
C	3	4	0	5	6	7	4
D	7	6	5	0	2	4	3
E	3	7	6	2	0	5	6
F	6	3	7	4	5	0	4
G	8	5	4	3	6	4	0

Encoding for PMX

$$A = 1,\ B = 2,\ C = 3,\ D = 4,\ E = 5,\ F = 6,\ G = 7$$

Fitness

$$L(\pi)$$

Objetive

$$\text{minimize } L(\pi)$$

Note

TSP fitness is a sum along the permutation edges.

Initialization: Population (Size 4) + Fitness

Population $P^{(0)}$ (minimize $L(\pi)$)

- P_1 : $[1, 2, 3, 4, 5, 6, 7]$ $L = 32$
- P_2 : $[1, 3, 2, 5, 4, 7, 6]$ $L = 29$
- P_3 : $[2, 6, 4, 5, 1, 3, 7]$ $L = 24$ $\leftarrow$ best
- P_4 : $[3, 1, 5, 2, 7, 4, 6]$ $L = 32$

Best tour P_3

2	6	4	5	1	3	7

$L = 24$

Permutation
(no repeats)

Note

Lower tour length = better fitness (minimization).

Selection: Tournament (Size 2) — Visual

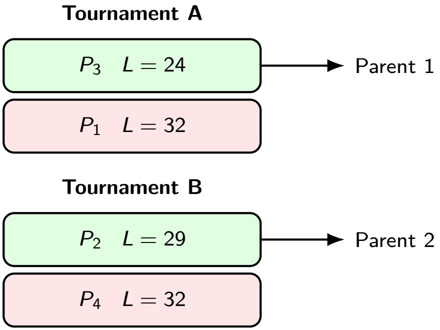
Assume two tournaments:

- Tournament A: P_3 vs $P_1 \Rightarrow$ winner P_3
- Tournament B: P_2 vs $P_4 \Rightarrow$ winner P_2

So parents are:

$$\text{Parent } 1 = P_3 = [2, 6, 4, 5, 1, 3, 7]$$

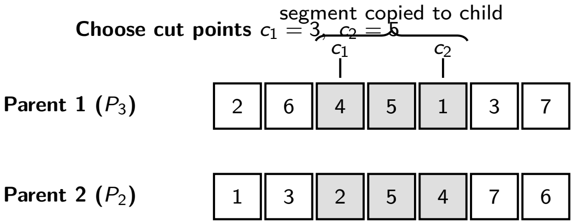
$$\text{Parent } 2 = P_2 = [1, 3, 2, 5, 4, 7, 6]$$



Note

Selection biases toward short tours, but keeps diversity.

PMX Step 1: Choose Cut Points and Mark Segments



Note

PMX copies a contiguous segment (positions) from Parent 1.

PMX Step 2: Copy Segment to Child + Build the Mapping

Copy the segment from P_1 (positions 3..5) into the child:

$$C = [\,\_, \_, \boxed{4, 5, 1}, \_, \_, \_, \_\,]$$

Build the PMX mapping from the swapped blocks (value-to-value):

$$P_1[3..5] = [4, 5, 1], \qquad P_2[3..5] = [2, 5, 4]$$

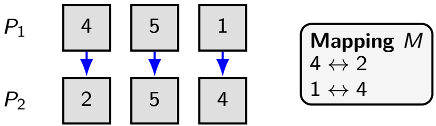
$$M : \quad 4 \leftrightarrow 2, \qquad 1 \leftrightarrow 4$$

($5 \leftrightarrow 5$ is trivial and can be ignored.)

Note

The copied segment fixes positions; the segment pairs define the repair mapping.

Swapped blocks (pos 3..5)



ignore $5 \leftrightarrow 5$

How it is used (repair intuition):

- When we try to place a value from P_2 outside the segment,
- if it **duplicates** something already in the child,
- we **follow the mapping** M (possibly chaining) until we get a value not in the segment.

PMX Step 3: Fill Remaining Positions from Parent 2 (with Repair)

Child after copy:

$$C = [\_, \_, 4, 5, 1, \_, \_]$$

Copied set $S = \{4, 5, 1\}$.

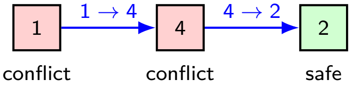
Now fill positions $1, 2, 6, 7$ from Parent 2: $[1, 3, \_, \_, \_, 7, 6]$

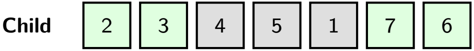
Key conflict (example at pos 1):

- try $1 \Rightarrow$ conflict $(1 \in S)$
- map $1 \to 4$ (still conflict)
- map $4 \to 2$ (safe) $\Rightarrow$ place 2

Then place remaining safe values in order:

$$C = [2, 3, 4, 5, 1, 7, 6]$$





Mutation: Swap Two Positions (Optional Diversification)

Assume swap mutation triggers once.

Before mutation:

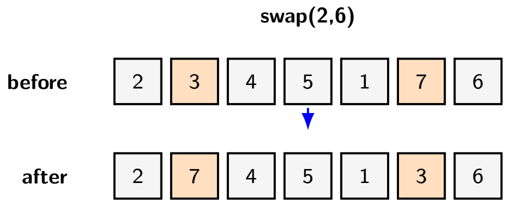
$$C = [2, 3, 4, 5, 1, 7, 6]$$

Swap positions 2 and 6:

$$C' = [2, 7, 4, 5, 1, 3, 6]$$

Interpretation (letters):

$$[B, G, D, E, A, C, F]$$



Note

Swap mutation preserves permutation feasibility and injects diversity.

Offspring Fitness + Replacement (Elitism)

Evaluate mutated offspring C':

$$C' = [2,7,4,5,1,3,6] \quad \Rightarrow \quad L(C') = 26$$

Elitist replacement (keep best 1):

$$\text{Keep } P_3 \ (L = 24)$$

Then fill remaining slots with best available:

$$\{P_3(24),\ C'(26),\ P_2(29),\ P_1(32)\}$$

Drop worst: $P_4(32)$.

Next population

elite: P_3 (24)
offspring: C' (26)
P_2 (29)
P_1 (32)
drop: P_4 (32)

Note

Elitism prevents losing the best tour while still exploring via offspring.