Ex6) Quaternion for 2D rotation

- Find quaternions for rotation about x-axis by θ
- Similarly, find quaternions for rotating about z-axis by $\theta = 0, 45^o, 90^o, 180^o$, etc.

Rotation about x-axis :

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \rightarrow \begin{aligned} r_{11} &= 1 \\ r_{12} &= r_{13} = r_{21} = r_{31} = 0 \\ r_{22} &= r_{33} = \cos\theta \\ -r_{23} &= +r_{32} = \sin\theta \end{aligned}$$

$$q_r = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}} = \frac{1}{2}\sqrt{1 + 1 + \cos\theta + \cos\theta} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{2\cos^2\frac{\theta}{2}}{2}} = \cos\frac{\theta}{2}$$

$$q_2 = q_3 = 0$$

$$q_1 = \frac{1}{4q_r}(r_{32} - r_{23}) = \frac{2\sin\theta}{2 \cdot 4q_r} = \frac{\sin\theta}{\sqrt{2(1+\cos\theta)}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sqrt{2\times 2\cos^2\frac{\theta}{2}}} = \sin\frac{\theta}{2}$$

$$q^T = \begin{bmatrix} \sin\frac{\theta}{2} & 0 & 0 & \cos\frac{\theta}{2} \end{bmatrix}$$

Rotation about z-axis:

$$q^T = \begin{bmatrix} 0 & 0 & \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$q(z, 90^\circ) = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$q(z, 45^\circ) = \begin{bmatrix} 0 & 0 & \frac{1}{2\sqrt{2+\sqrt{2}}} & \frac{\sqrt{2+\sqrt{2}}}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.3827 & 0.3827 \end{bmatrix}$$

$$q(z, 180^\circ) = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$