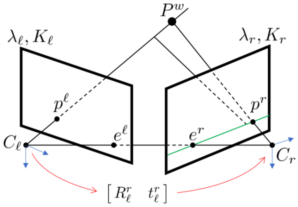
Ex3) Epipolar Geometry

- Assume C_ℓ is the world frame (i.e. $\lambda_\ell \bar{p}^\ell = K_\ell \,[\, I \mid 0 \,]\, \bar{P}^w$).
  Find the epipole e^r and the epipolar line (green) on the image plane of the right camera using parameters given.



Camera equation for left camera C_ℓ

$$\lambda_\ell \bar{p}^\ell = K_\ell \,[I \mid 0]\, \bar{p}^w = K_\ell \,[I \mid 0] \begin{bmatrix} p^w \\ 1 \end{bmatrix} = K_\ell \, p^w$$

$$\Rightarrow \quad p^w = \lambda_\ell K^{-1} \bar{p}^\ell \quad (1)$$

Camera equation for right camera C_r

$$\lambda_r \bar{p}^r = K_r \left[R_\ell^r \mid t_\ell^r \right] \bar{p}^w = K_r \left(R_\ell^r p^w + t_\ell^r \right) \quad (2)$$

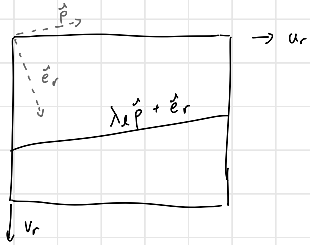
Substituting (1) to (2) :

$$\lambda_r \bar{p}^r = K_r \left(R_\ell^r \lambda_\ell K_\ell^{-1} \bar{p}^\ell + t_\ell^r \right)$$
$$= \lambda_\ell K_r R_\ell^r K_\ell^{-1} \bar{p}^\ell + K_r t_\ell^r$$

- For a fixed p^ℓ, varying λ_ℓ corresponds to moving on the epipolar line on the C_r. (green line)
- $\lambda_\ell = 0$ corresponds to zero depth on C_ℓ, thus the origin of C_ℓ, and the corresponding point, i.e. $K_r t_\ell^r$ coincides with e^r, i.e. the epipole on C_r.

$\therefore$ The epipole $e^r = K_r t_\ell^r$

The epipolar line on C_r $= \lambda_\ell \underbrace{K_r R_\ell^r K_\ell^{-1} \bar{p}^\ell}_{\hat{p}} + \underbrace{K_r t_\ell^r}_{\hat{e}^r}$



For rectified stereo vision, $R_l^r = I$, $t_l^r = \begin{bmatrix} -b \\ 0 \\ 0 \end{bmatrix}$

$K_l = K_r = K$ with identical cameras

Horizontal to the epipolar plane, $\lambda_l = \lambda_r = \lambda$, $v^l = v^r$.

$$\Rightarrow \lambda \bar{p}^r = \lambda \bar{p}^l + \underbrace{\begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{K} \begin{bmatrix} -b \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \lambda \bar{p}^r = \lambda \bar{p}^l + \begin{bmatrix} -b\alpha_u \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \lambda \begin{bmatrix} u^r \\ v^r \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} u^l \\ v^l \\ 1 \end{bmatrix} + \begin{bmatrix} -b\alpha_u \\ 0 \\ 0 \end{bmatrix} \Rightarrow \lambda (u^r - u^l) = -b\alpha_u$$

$$\lambda = b \frac{f}{u^l - u^r} \quad \text{(in pixels)}$$