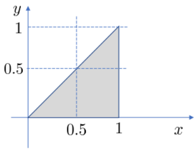
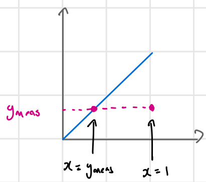
Ex5) MMSE Estimation

- (x, y) are uniformly distribute on the triangle, i.e. the joint pdf is given by $p(x,y) = \begin{cases} 2 & \text{if } (x,y) \in A \\ 0 & \text{otherwise} \end{cases}$
- We measured y, and it came out $y_{meas} = 0.3$, what is the best estimate of x in the MMSE sense, i.e. $\hat{x}_{MMSE}$?



Given $y = y_{meas}$, the x value is uniformly distributed on the horizontal line segment from y_{meas} to 1



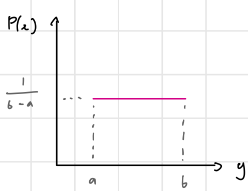
Then, the mean of x is the middle point of the segment:

$$E[x \mid y = y_{meas}] = \frac{1 + y_{meas}}{2}$$

$$\boxed{\hat{x}_{MMSE} = \frac{1 + y_{meas}}{2}}$$

We can calculate the MSE: $E[(x - \hat{x})^2 \mid y = y_{meas}] = \frac{1}{12}(1 - y_{meas})^2$

Variance of uniform distribution:



$$\mu = E[x] = \frac{b+a}{2}$$

$$\sigma = E[(x-\mu)^2] = \int_a^b p(x)\,(x-\mu)^2\,dx$$

$$= \int_a^b \frac{1}{b-a}\left(x - \frac{a+b}{2}\right)^2 dx$$

$$= \frac{1}{b-a}\,\frac{1}{3}\left[\left(x - \frac{a+b}{2}\right)^3\right]_a^b$$

$$= \frac{1}{3(b-a)}\left(\left(\frac{b-a}{2}\right)^3 - \left(\frac{b-a}{2}\right)^3\right)$$

$$= \frac{1}{3(b-a)}\left(2\left(\frac{b-a}{2}\right)^3\right) = \frac{(b-a)^{12}}{12}$$