

Ex3) Drug Testing

- Consider a drug test procedure that has
 - 99% of correct positive; the test will correctly catch 99 times out of 100 drug users
 - 98% of correct negative; the test will indicate negative 98 times out of 100 clean subjects (non-drug users)
 - 0.5% of residents are known to be drug users
- We took this test on a randomly selected citizen, and the result came out positive
- What is the probability that this person was, indeed, a drug user?



Event A : A person tested positive

Event B : A person used the drug

$$1) P(A|B) = 0.99$$

$$2) P(A^c|B^c) = 0.98$$

$$3) P(B) = 0.005$$

$$P(B|A) = \frac{P(A|B) P(B)}{P(A)} = \frac{P(A|B) P(B)}{\sum_B P(A|B)} = \frac{P(A|B) P(B)}{P(A|B) P(B) + P(A|B^c) P(B^c)}$$

$$P(A|B^c) P(B^c) = (1 - P(A^c|B^c)) (1 - P(B)) = (1 - 0.98) (1 - 0.005) = 0.02 \times 0.995$$

$$= \frac{0.99 \cdot 0.005}{0.99 \cdot 0.005 + 0.02 \cdot 0.995} = 0.112 \approx 11.2\%$$