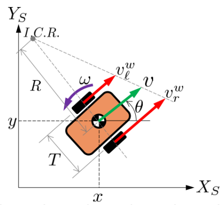
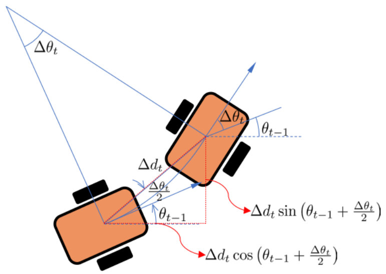
Ex1) Odometry with Wheel Encoder

- Assume that the 2W differential drive robot is equipped with encoders at each wheel. Derive equations for odometry.





Δd_t^l : Incremental distance traveled by left wheel at time t

Δd_t^r : Incremental distance traveled by right wheel at time t

$$\Delta d_t^l = \left(R - \frac{T}{2}\right)\Delta\theta_t, \qquad \Delta d_t^r = \left(R + \frac{T}{2}\right)\Delta\theta_t$$

$$\Rightarrow \quad d_t = R\Delta\theta_t = \frac{d_t^l + d_t^r}{2}, \qquad \Delta\theta_t = \frac{\Delta d_t^r - \Delta d_t^l}{T}$$

$$\Longrightarrow \begin{bmatrix} \Delta d_t \\ \Delta\theta_t \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{T} & -\frac{1}{T} \end{bmatrix} \begin{bmatrix} \Delta d_t^r \\ \Delta d_t^l \end{bmatrix}$$

From the figure :

$$x_t = x_{t-1} + \Delta d_t \cos\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right)$$

$$y_t = y_{t-1} + \Delta d_t \sin\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right)$$

$$\theta_t = \theta_{t-1} + \Delta\theta_t$$

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \cos\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right) & 0 \\ \sin\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta d_t \\ \Delta\theta_t \end{bmatrix}$$

$$= \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\cos\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right) & \frac{1}{2}\cos\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right) \\ \frac{1}{2}\sin\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right) & \frac{1}{2}\sin\left(\theta_{t-1} + \frac{\Delta\theta_t}{2}\right) \\ \frac{1}{T} & -\frac{1}{T} \end{bmatrix} \begin{bmatrix} \Delta d_t^r \\ \Delta d_t^l \end{bmatrix}$$