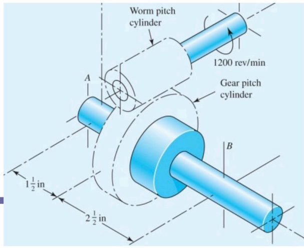
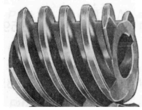
Ex 9)

- A 2-tooth right-hand worm transmits 1 hp at 1200 rpm to a 30-tooth worm gear. The diametral pitch of the gear is 6 teeth/in. The pitch dia. of the worm is 2 in. The normal pressure angle $\phi_n = 14.5^o$. Friction coefficient $\mu = 0.03$. Compute a) the forces on the output shaft bearings, b) efficiency, c) output power at the gear.



Example of 2-tooth worm

$L = 2p_x \longrightarrow \tan\lambda = \frac{L}{\pi d_W} = \frac{2p_x}{\pi d_W}$



(right-hand worm)

Important point to note : Power is given at the input side ⟹ Can find forces at worm.

$$T_{in} = \frac{P_{in}}{w_{in}} = \frac{1 \times 550 \text{ lb}\cdot\text{ft/s}}{1200 \times \frac{2\pi}{60}} = 4.3768 \text{ lb}\cdot\text{ft}$$

$$W_{tW} = \frac{T_{in}}{d_w/2} = \frac{4.3768 \text{ lb}\cdot\text{ft}}{\left(\frac{2\text{ in}}{2}\right)\cdot\frac{1\text{ ft}}{12\text{ in}}} = 52.5 \text{ lb}$$

$$pP = \pi \rightarrow p_x = \frac{\pi}{P} = \frac{\pi}{6} = 0.5236 \text{ in}$$

↳ P = diametrical pitch $= \frac{N_G}{d_G}$

Gear rotates by 1 tooth when the worm moves by p_x ⟹ $p_x = p_G$

To compute W_{xW}, W_{rW} from W_{tW}, we need

$$\lambda = \tan^{-1}\left(\frac{L}{\pi d_w}\right) = \tan^{-1}\left(\frac{N_w\, p_x}{\pi d_w}\right) = \tan^{-1}\left(\frac{2 \times 0.5236}{\pi \cdot 2}\right) = 9.46^\circ$$

With λ, we can :

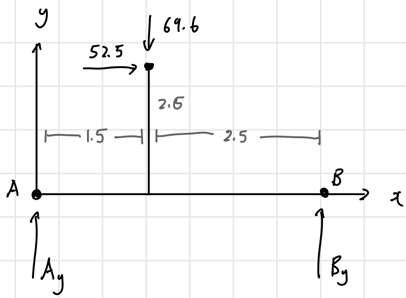
$$W_{xW} = W_{tW}\,\frac{\cos\phi_n\cos\lambda - \mu\sin\lambda}{\cos\phi_n\sin\lambda + \mu\cos\lambda} = 52.5 \times \frac{\cos 14.5^\circ \cos 9.46^\circ - 0.03 \sin 9.46^\circ}{\cos 14.5^\circ \sin 9.46^\circ + 0.03\cos 9.46^\circ}$$

$$= 264.4 \text{ lbf}$$

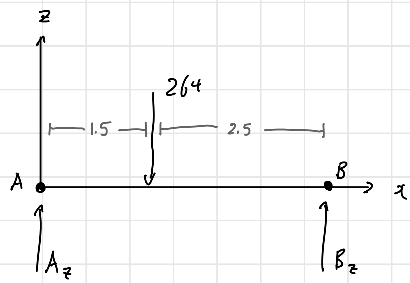
$$W_{rW} = W_{tW} \frac{\sin\phi_n}{\cos\phi_n \sin\lambda + \mu\cos\lambda} = 69.6 \text{ lbf}$$

$$\Rightarrow \begin{cases} W_{tG} = W_{xW} = 264.4 \text{ lbf} \\ W_{rG} = W_{rW} = 69.6 \text{ lbf} \\ W_{xG} = W_{tW} = 52.5 \text{ lbf} \end{cases}$$

x-y plane



x-z plane



$$\Sigma F_y = A_y + B_y - 69.6 = 0$$

$$\Sigma M_A = -(1.5)(52.5) - (2.5)(69.6) + (4)(B_y) = 0$$

$$\Rightarrow B_y = 58.9 \text{ lbf}, \quad A_y = 10.7 \text{ lbf}$$

$$\Sigma F_z = A_z - 264.4 + B_z = 0$$

$$\Sigma M_A = -(1.5)(264) + (4)(B_z) = 0$$

$$\Rightarrow B_z = 99 \text{ lbf}, \quad A_z = 165 \text{ lbf}$$

$$\Sigma F_x = B_x + 52.5 = 0$$

$$\Rightarrow B_x = -52.5 \text{ lbf}$$

Bearing forces

$$F_{rA} = \sqrt{(A_y)^2 + (A_z)^2} = 165.35 \text{ lbf} \qquad F_{aA} = 0$$

$$F_{rB} = \sqrt{(B_y)^2 + (B_z)^2} = 115.2 \text{ lbf} \qquad F_{aB} = 52.5 \text{ lbf}$$

Torsion on shaft : $T = 264 \times 2.5 = 660 \text{ lbf} \cdot \text{in}$

Efficiency :

$$\eta = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cos\lambda} = 0.8388 \rightarrow \boxed{84\%} \quad (b)$$

Then output power $P_{out} = \eta \cdot P_{in} = \boxed{0.84 \text{ hp}}$ (c)