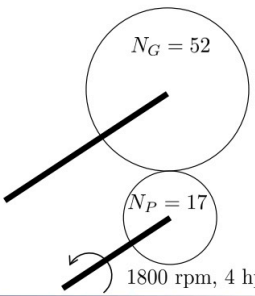
Ex 12) Strength Analysis: Spur Gear

Compute safety factors of the pinion in bending and contact wear (pitting).

- **Assume uniform loading ($K_o = 1$), load distribution factor $K_m = 1.22$, size factor $K_s = 1$, rim thickness factor $K_B = 1$ and surface cond. factor $C_f = 1$.**
- **Use pinion life of 10^8 cycles, reliability $R = 0.9$, stress cycle factor $Y_N = 1.3558N^{-0.0178}$ (bending), $Z_N = 1.4488N^{-0.023}$ (pitting) and $C_H = 1.0$.**
- **Use $S_t = 77.3H_B + 12800$ psi (bending) and $S_c = 322H_B + 29100$ psi (pitting)**

Parameters	Pinion	Gear
P (diametral pitch)	10 teeth/in	
F (face width)	1.5 in	
Q_v (quality number)	6	
Material	Grade 1 steel	
E Young's modulus	$30(10^6)$ psi	
ν Poisson's ratio	0.3	
H_B (Brinell hardness)	240	200
J (bending geo. factor)	$J_P = 0.3$	$J_G = 0.4$
I (pitting geo. factor)	$I = 0.121$	



AGMA equations for spur gears (for bending)

$$P = \frac{N}{d} \rightarrow d_p = \frac{N}{P} = \frac{17}{10} = 1.7$$

$$\sigma = K_o K_v K_s K_m K_B \cdot \frac{W_t P}{FJ}$$

$P = 10$, $F = 1.5$, $J_P = 0.3$, $K_s = K_o = K_B = 1$, $K_m = 1.22$

$$W_t = \frac{T_i}{d_p/2} = \frac{P_i / n_p}{d_p/2} = \frac{2P_i}{n_p d_p} = \frac{2 \times 4 \times 550}{\left(1800 \times \frac{2\pi}{60}\right) \times \left(1.7 \times \frac{1}{12}\right)} = 164.8 \text{ lbf}$$

(power, speed)

Computing factors:

$$K_v \text{ (dynamic factor)} = \left(\frac{A + \sqrt{V}}{A}\right)^B,$$

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$\therefore A = 50 + 56(1 - 0.8255) = 59.77$$

$$V \text{ (pitch line velocity)} = \pi d_p \cdot n_p = \pi \times \left(1.7 \times \frac{1}{12}\right) \times 1800 = 801.1 \text{ ft/min}$$

$$\Rightarrow K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

$$\sigma_p = K_o K_v K_s K_m K_B \cdot \frac{W_t P}{F J}$$

$$= 1 \times 1.377 \times 1 \times 1.22 \times 1 \times \frac{164.8 \times 10}{1.5 \times 0.3} = 6152 \text{ psi}$$

$$(\sigma_p)_{all} = \frac{Y_N}{S_F K_T K_R} S_t$$

$K_T = 1 \ (< 250°)$

$K_R = 0.658 - 0.0759 \ln(1-R) = 0.833$

$Y_N = 1.3558(N)^{-0.0178} = 1.3558 \times (10^6)^{-0.0178} = 0.977$

$S_t = 77.34 H_B + 12800 = 77.3 \times 240 + 12800 = 31352 \text{ psi}$

$$S_F = \frac{\frac{Y_N S_t}{K_T K_R}}{\sigma_p} = \frac{\frac{0.977 \times 31352}{1 \times 0.833}}{6152} = \boxed{5.98}$$

AGMA equations for spur gears (for contact / wear)

$$\sigma_c = C_p \sqrt{K_o K_v K_s K_m C_f \cdot \frac{W_t}{F d_p I}}, \quad C_f = 1, \quad I = 0.121$$

The elastic coefficient ($\nu_p = \nu_g = 0.3$, $E_p = E_g = 30 \times 10^6$ psi)

$$(\sigma_c)_p = 2291 \times \sqrt{1 \times 1.377 \times 1 \times 1.22 \times 1 \times \frac{164.8}{1.5 \times 1.7 \times 0.121}} = 68626 \text{ psi}$$

$$(\sigma_c)_{all} = \frac{Z_N C_H}{S_F K_T K_R} S_c$$

$Z_N = 1.4488 (N)^{-0.023} = 1.4488 (10^6)^{-0.023} = 0.948$

$S_c = 322 H_B + 29100 = 322 \times 240 + 29100 = 106400 \text{ psi}$

$$S_F = \frac{\frac{Z_N C_H S_c}{K_T K_R}}{(\sigma_c)_p} = \frac{\frac{0.948 \times 1 \times 106400}{1 \times 0.833}}{68626} = \boxed{1.76}$$

$(S_F)_{bending} = 5.98$ and $(S_F)_{contact} = 1.76$