Ex 11) Spur Gear Strength: Pitting

- **The pinion in Ex 10) is to be mated with the following gear. Estimate the factor of safety based on the surface fatigue failure.**

(ASTM No. 50 cast iron with $S_c = 0.32H_B$ kpsi for 10^8 cycles.)
$N = 50$ teeth, $F = 38$ mm (face width), $m = 3$ (module), $\phi = 20^o$,
manufactured by cutting

$$\sigma_c = -C_p \left[\frac{K_v W_t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}, \qquad C_p = \left[\frac{1}{\pi \left(\frac{1-\nu_p^2}{E_p} + \frac{1-\nu_a^2}{E_a} \right)} \right]$$

Tangential force is same as the pinion $\Rightarrow$ $W_t = 1552$ N

From A-5, pinion is cast iron and gear is carbon steel

Table A–5

Physical Constants of Materials

Material	Modulus of Elasticity E Mpsi	Modulus of Elasticity E GPa	Modulus of Rigidity G Mpsi	Modulus of Rigidity G GPa	Poisson's Ratio ν
Aluminum (all alloys)	10.4	71.7	3.9	26.9	0.333
Beryllium copper	18.0	124.0	7.0	48.3	0.285
Brass	15.4	106.0	5.82	40.1	0.324
Carbon steel	30.0	207.0	11.5	79.3	0.292
Cast iron (gray)	14.5	100.0	6.0	41.4	0.211
Copper	17.2	119.0	6.49	44.7	0.326

$E_p = 207$ GPa, $\nu_p = 0.292$ (carbon steel)
$E_a = 100$ GPa, $\nu_a = 0.211$ (cast iron)

$$C_p = \left[\frac{1}{\pi \left(\frac{1-0.292^2}{207 \times 10^9} + \frac{1-0.211^2}{100 \times 10^9} \right)} \right]^{1/2} = 150.9 \times 10^3 \ [\text{Pa}^{1/2}]$$

$$r_1 = \frac{d_p \sin\phi}{2} = \frac{m \cdot N \sin\phi}{2} = \frac{3 \cdot 16 \cdot \sin 20^\circ}{2} = 8.2 \text{ mm}$$

(from Example 10)

$$r_2 = \frac{d_a \sin\phi}{2} = \frac{3 \cdot 50 \sin\phi}{2} = 25.7 \text{ mm}$$

$$\sigma_c = -(150.9 \times 10^3) \times \left[\frac{1.5 \times 1652}{38 \times 10^{-3} \times \cos 20^\circ} \left(\frac{1}{8.2 \times 10^{-3}} + \frac{1}{25.7 \times 10^{-3}} \right) \right]^{\frac{1}{2}} = -488.68 \times 10^6 \text{ Pa}$$

(from Example 10)

Allowable strength (reference)

Table A–24

Mechanical Properties of Three Non-Steel Metals

(a) Typical Properties of Gray Cast Iron

[The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 20 kpsi. Note particularly that the tabulations are *typical* of several heats.]

ASTM Number	Tensile Strength S_{utt} kpsi	Compressive Strength S_{ucr} kpsi	Shear Modulus of Rupture S_{sur} kpsi	Modulus of Elasticity, Mpsi		Endurance Limit* S_{er} kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
				Tension†	Torsion			
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

$$S_c = 0.32\,H_B = 0.32 \times 262\ \text{kpsi} = 83.84\ \text{kpsi}$$

$$= 83.84 \times 6.9\ \text{MPa} = 578.5$$

The safety factor:

$$n = \frac{S_c}{|\sigma_c|} = \frac{578.5}{488.68} = 1.18$$