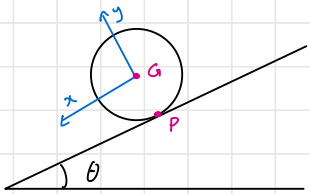
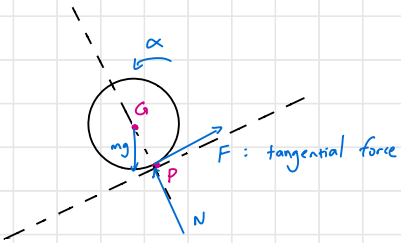
Example: Rolling cylinder with no slipping / bouncing, starting from rest. Find a_G and α.



- No bouncing → $a_{Gy} = 0$

$$\sum f_x = ma_{Gx}$$
$$mg\sin\theta - F = ma_{Gx}$$
$$F = mg\sin\theta - ma_{Gx}$$

$$\sum f_y = ma_{Gy}$$
$$N - mg\cos\theta = 0$$



Using moment about G

$$Fr = I_G\alpha$$
$$mg\sin\theta\, r - ma_{Gx}r = I_G\alpha$$
$$ma_{Gx} = mg\sin\theta - \frac{I_G\alpha}{r}$$

No slip → $a_{Gx} = r\alpha$

$$\alpha = \frac{a_{Gx}}{r}$$

$$ma_{Gx} = mg\sin\theta - \frac{I_G a_{Gx}}{r^2}$$
$$ma_{Gx}r^2 = mgr^2\sin\theta - I_G a_{Gx}$$
$$(mr^2 + I_G)\,a_{Gx} = mgr^2\sin\theta$$
$$a_{Gx} = \frac{mgr^2\sin\theta}{mr^2 + I_G}$$

I_G for cylinder $= \frac{mr^2}{2}$

$$a_{Gx} = \frac{mgr^2\sin\theta}{\frac{3}{2}mr^2}$$

$$\boxed{a_{Gx} = \frac{2}{3}g\sin\theta}$$

Using moment about P

$$M_P = I_G\alpha + ma_{Gx}r \quad (M_P = mg\sin\theta\, r,$$
$$mg\sin\theta\, r = I_G\alpha + ma_{Gx}r \quad F \text{ goes through point } P \therefore \text{ no moment from } F)$$
$$\alpha = \frac{a_{Gx}}{r}$$
$$mg\sin\theta\, r = I_G\frac{a_{Gx}}{r} + ma_{Gx}r$$
$$I_G = \frac{mr^2}{2}$$
$$mg\sin\theta\, r = \frac{mr^2}{2}\frac{a_{Gx}}{r} + ma_{Gx}r$$
$$mg\sin\theta = \frac{m}{2}a_{Gx} + ma_{Gx}$$
$$mg\sin\theta = \frac{3}{2}m(a_{Gx})$$

$$\boxed{a_{Gx} = \frac{2}{3}g\sin\theta}$$

What if the cylinder slips?

$F \le F_{friction}$ → no slipping

$F > F_{friction}$ → slipping

$$F_{friction} = \mu_s N = \mu_s mg\cos\theta$$
$$F = mg\sin\theta - ma_{Gx} = mg\left(\sin\theta - \tfrac{2}{3}\sin\theta\right)$$
$$= \frac{1}{3}mg\sin\theta$$

No slip: $F \le F_{friction}$

$$\frac{1}{3}mg\sin\theta \le \mu_s mg\cos\theta$$
$$\frac{1}{3}\sin\theta \le \mu_s\cos\theta$$

$$\boxed{\tan\theta \le 3\mu_s}$$