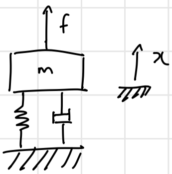
Example 1



$$m\ddot{x} + c\dot{x} + kx = f$$

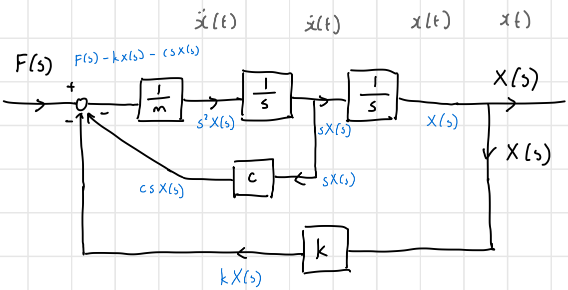
$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

$$\ddot{x}(t) + \frac{c\dot{x}}{m} + \frac{k}{m}x(t) = \frac{f(t)}{m}$$

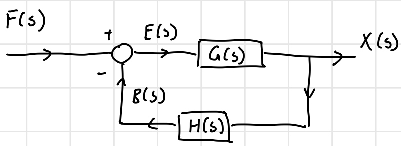
$$s^2X(s) + \frac{c}{m}sX(s) + \frac{k}{m}X(s) = \frac{F(s)}{m}$$

$$s^2X(s) = \frac{F(s)}{m} - \frac{c}{m}sX(s) + \frac{k}{m}X(s)$$

$$X(s) = \frac{1}{m}\left[\frac{F(s)}{s^2} - \frac{1}{s}X(s) - \frac{k}{s^2}X(s)\right]$$



Example 2 : Feedback Loop



$$B(s) = X(s)\,H(s)$$

$$E(s) = F(s) - B(s)$$

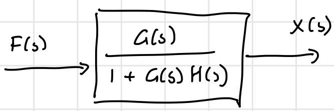
$$X(s) = E(s)\,G(s) \quad \text{or} \quad E(s) = \frac{X(s)}{G(s)}$$

$$\frac{X(s)}{G(s)} = F(s) - X(s)\,H(s)$$

$$\frac{X(s)}{G(s)} + X(s)\,H(s) = F(s)$$

$$\left[\frac{1}{G(s)} + H(s)\right] X(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{\frac{1}{G(s)} + H(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

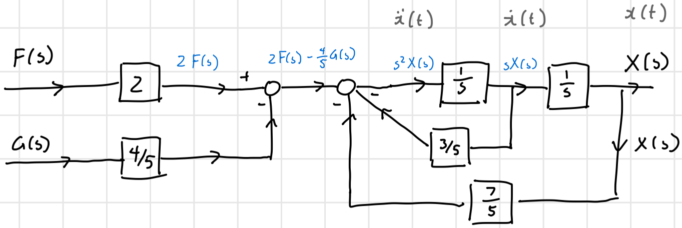


Example 3: P 5.4

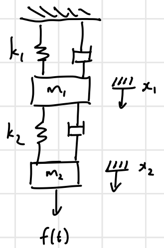
$$5\ddot{x} + 3\dot{x} + 7x = 10f_t - 4g(t)$$

Output: $X(s)$

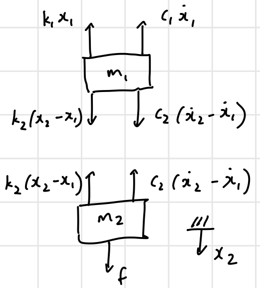
$$\ddot{x} = \frac{10}{5}f(t) - \frac{4}{5}g(t) - \frac{3}{5}\dot{x} - \frac{7}{5}x$$

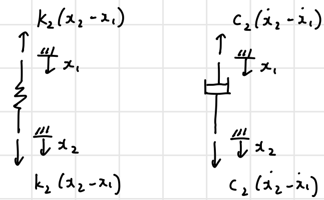


Example: 2 D.O.F



Assuming $x_1 > x_2$ and $\dot{x}_1 > \dot{x}_2$:





$$\begin{cases} m_1\ddot{x}_1 = k_2(x_2-x_1) + c_2(\ddot{x}_2-\ddot{x}_1) + c_2(\dot{x}_2-\dot{x}_1) - k_1x_1 - c_1\dot{x}_1 \\ m_2\ddot{x}_2 = f - k_2(x_2-x_1) - c_2(\dot{x}_2-\dot{x}_1) \end{cases}$$

$$\begin{cases} m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1+k_2)x_1 - k_2x_2 = 0 \\ m_2\ddot{x}_2 + c_2\dot{x}_2 - c_2\dot{x}_1 - k_2x_1 + k_2x_2 = f \end{cases}$$

$$\ddot{x}_1 = \frac{1}{m_1}\left[k_2(x_2-x_1) + c_2(\dot{x}_2-\dot{x}_1) - k_1x_1 - c_1\dot{x}_1\right]$$
$$\ddot{x}_2 = \frac{1}{m_2}\left[f - k_2(x_2-x_1) - c_2(\dot{x}_2-\dot{x}_1)\right]$$