Example 6-6

In a 240-V shunt DC motor, armature resistance is $R_a = 0.3\ \Omega$. Under load, armature current is $I_a = 20\ A$ and motor speed is $n = 600\ rpm$. The field flux is suddenly decreased by 2%. Find

(a) the ratio of the developed torque right after the change in the field flux to the developed torque before the change.

(b) the new steady-state speed.

Note: The load torque remains the same while field flux is changed.

a. $\tau_{m,old} = K\phi_{old} I_{a,old} = K\phi_{old} \times 20 = 20 K\phi_{old}$

$E_{c,old} = V_L - I_{a,old} R_a = 240 - 20 \times 0.3 = 234\ V$

Speed cannot change instantly due to inertia, but $E_c = K'\phi n$ changes due to change in ϕ.

$$\frac{E_{c,new}}{E_{c,old}} = \frac{K'\phi_{new} n_{new}}{K'\phi_{old} n_{old}} = \frac{\phi_{new}}{\phi_{old}} = 0.98$$

$E_{c,new} = 0.98\, E_{c,old} = 0.98 \times 234 = 229.32\ V$

$$I_{a,new} = \frac{V_L - E_{c,new}}{R_a} = \frac{240 - 229.32}{0.3} = 35.6\ A$$

$\tau_{m,new} = K\phi_{new} I_{a,new} = K \times 0.98\, \phi_{old} \times 35.6 = 34.89\ K\phi_{old}$

$$\frac{\tau_{m,new}}{\tau_{m,old}} = \frac{34.89\ K\phi_{old}}{20\ K\phi_{old}} = 1.75$$

b. Load torque hasn't changed $\rightarrow$ $\tau_{L,\text{new ss}} = \tau_{L,\text{old ss}}$

Ignore mechanical, core and miscellaneous losses,

$$\tau_m = \tau_L$$

$$\therefore \tau_{m,\text{new steady-state}} = 0.98\,\phi_{\text{old, steady-state}}$$

$$\tau_{m,\text{new steady-state}} = K\phi_{\text{new}} I_{a,\text{new steady-state}}$$

$$= K \times 0.98\,\phi_{\text{old}} I_{a,\text{new steady-state}}$$

$$= 0.98\,I_{a,\text{new steady-state}} \times K\phi_{\text{old}} = 20(K\phi_{\text{old}})$$

$$I_{a,\text{new steady state}} = \frac{20}{0.98} = 20.41\text{ A}$$

To find the new steady-state speed, we can write:

$$E_{c,\text{new steady-state}} = V_L - I_{a,\text{new ss}} R_a = 240 - 20.41 \times 0.3 = 233.88\text{ V}$$

Then, the ratio of the new steady-state condition to the old steady-state condition:

$$\frac{E_{c,\text{new ss}}}{E_{c,\text{old ss}}} = \frac{K'\phi_{\text{new, ss}}\, n_{\text{new, ss}}}{K'\phi_{\text{old, ss}}\, n_{\text{old, ss}}}$$

$$= \frac{K' \times 0.98\,\phi_{\text{old, ss}} \times n_{\text{new, ss}}}{K' \times \phi_{\text{old}} \times 600}$$

$$\therefore \frac{233.88}{234} = \frac{0.98}{600}\, n_{\text{new, ss}}$$

$$n_{\text{new, ss}} = \boxed{611.93\text{ RPM}} > 600\text{ RPM}$$