Example 6-3

Consider the single-loop DC machine of Fig. E6-3. The power source is a 48-V battery. The internal resistance of the battery plus the resistance of the coil is $R = 0.1\ \Omega$. The length of the loop is $l=0.5\ m$ and its radius is $r=25\ cm$. The magnetic field density under each pole face is $B=0.5\ T$. Initially, no load is connected to the shaft of the machine. When the battery is connected to the loop through the commutator, find:

(a) the starting current,
(b) the starting torque, and
(c) the no-load steady-state speed if no-load steady-state armature current is assumed to be equal to zero.

If a mechanical load of $5\ N.m$ is connected to the loop, find:

(d) the new steady-state speed, and
(e) the mode of operation of the machine (motor or generator).

If at no load, a torque of $5\ N.m$ is externally applied to the shaft in the direction of rotation, find:

(f) the new steady-state speed, and
(g) the mode of operation (motor or generator).

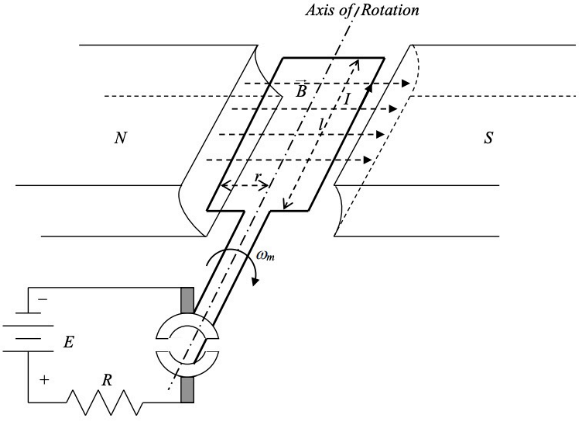

Fig. E6-3

$l = 0.5\ m$
$r = 0.25\ m$
$R = 0.1\ \Omega$
$B = 0.5\ T$
$E = 48\ V$

a. $I_a = \dfrac{E - E_c}{R}$

$E_c = 2vBl \rightarrow v = 0$ at startup $\rightarrow E_c = 0$

$I_a = \dfrac{E}{R} = \dfrac{48}{0.1} = \boxed{480\ A}$

Starting armature current > steady-state armature current due to lack of back emf to limit current.

b. Starting torque:

$$\tau_{start} = 2i\ell Br = 2(480)(0.5)(0.5)(0.25) = \boxed{60\ \text{N}\cdot\text{m}}$$

c. No-load steady-state $\rightarrow$ $I_a = 0$

$$\therefore\ E_c = E - I_a R = E = 48\ V$$

$$E_c = 2vB\ell = 2 \times \frac{2\pi r n}{60} \times B\ell = \frac{r\pi n B\ell}{15}$$

$$\therefore\ n = \frac{15\,E_c}{r\pi B\ell} = \frac{15 \times 48}{0.25 \times \pi \times 0.5 \times 0.5} = \boxed{3666.93\ \text{RPM}}$$

d. At steady-state, $\tau_m = \tau_L = 5\ \text{N}\cdot\text{m}$

$$\tau_m = 2 I_a \ell B r$$

$$I_a = \frac{\tau_m}{2\ell B} = \frac{5}{2 \times 0.5 \times 0.5 \times 0.25} = 40\ A$$

$$E_c = E - I_a R = 48 - 40 \times 0.1 = 44V$$

$$E_c = 2vB\ell = \frac{r\pi n B\ell}{15}$$

$$n = \frac{15 E_c}{r\pi B\ell} = \frac{15 \times 44}{0.25 \times \pi \times 0.5 \times 0.5} = \boxed{3361.35\ \text{RPM}}$$

e. The machine is a $\boxed{\text{motor}}$, driving a load of 5 N·m at 3361.35 RPM.
The speed drops from 3666.93 at no-load to 3361.35 due to increase in load.

f. Rotated by external 5 N·m $\rightarrow$ rotor will speed up since external torque is added to torque that is already generated.

$n\uparrow \Rightarrow E_c\uparrow$ until $E_c > E$ and I_a changes direction,
so that machine becomes generator.

$$\tau_c = \tau_m = 5\ N\cdot m$$

$$\tau_c = 2I_a l B r$$

$$I_a = \frac{\tau_c}{2lB} = \frac{5}{2\times 0.5\times 0.5\times 0.25} = 40A.$$ Opposite direction from d) and e)

$$E_{gen} - E = RI_a$$

$$E_{gen} = E + RI_a = 48 + 0.1(40) = 52V$$

$$E_{gen} = 2vBl = \frac{r\pi n Bl}{15}$$

$$n = \frac{15E_{gen}}{r\pi Bl} = \frac{15\times 52}{0.25\times\pi\times 0.5\times 0.5} = \boxed{3972.51\ RPM}$$

g. This machine is working as a $\boxed{\text{generator.}}$
Input torque produces mechanical power on shaft, which results in a current (and electric power) being forced into the source.