Example

You are designing a test rig that requires a temperature measurement with relatively fast response for a heat transfer coefficient of 400 W/m²K. To avoid a chemical effect, the tip the thermocouple must be stainless steel which has a relatively low thermal conductivity. You compromise in your design for a time constant, τ, of 1 s for your application.

$$L_c = \frac{V}{A} = \frac{\pi d^3/6}{\pi d^2} = \frac{d}{6}$$

Stainless steel at k = 20 W/mK, c_p = 400 J/kgK, and ρ = 8500 kg/m³

a) The thermocouple tip can be treated as a sphere. What is the maximum diameter of your thermocouple tip?
b) The thermocouple is at an initial temperature of 25°C and placed in a hot gas stream of 200°C. How long does it take for the thermocouple tip to reach 199°C?

a. $\tau = R_{th} C_{th}$

$= \left(\frac{1}{hA_s}\right)(\rho V c_p)$

$= \frac{d}{6}\left(\frac{\rho c_p}{h}\right)$

$d = \frac{6\tau h}{\rho c_p} = \frac{6 \times 1[s] \times 400[W/m^2K]}{8300[kg/m^3]\ 400[J/kgK]} = \boxed{0.706\ [mm]}$

b. $Bi = \frac{hL_c}{k} = \frac{h}{k} L_c = \frac{400[W/m^2K]}{20[W/mK]} \cdot \frac{7.06 \times 10^{-4}[m]}{6} = 2.35 \times 10^{-3} \leq 0.1$

∴ Lumped capacitance assumption holds

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-t/\tau}$$

$$-t/\tau = \ln\left(\frac{T(t) - T_\infty}{T_i - T_\infty}\right)$$

$$t = -1[s] \ln\left(\frac{199°C - 200°C}{25°C - 200°C}\right)$$

$= \boxed{5.16\ [s]}$