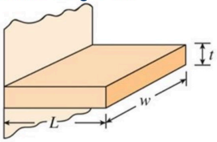
Example

A single straight brass fin with rectangular cross-section is 5 cm long.

The temperature at the base of the fin is 60°C and the surrounding ambient air is 30°C . The brass conducts heat at 110 W/mK and the convective heat transfer coefficient is 10 W/m²K.

Treating the fin with an adiabatic tip of corrected length, determine:

a) The rate of heat transfer from the fin.
b) Thermal resistance of the fin.
c) Efficiency of the single fin
d) Effectiveness of the single fin



$L = 5$ cm
$w = 10$ cm
$t = 1$ mm

$h = 10 \; W/m^2K$

$k = 110 \; W/mK$

$A_c = wt = (10 \times 10^{-2} \, m)(1 \times 10^{-3} \, m) = 10^{-4} \, [m]$

$P = 2(w + t) = 0.202 \, [m]$

$$\frac{A_c}{P} = \frac{wt}{2(w+t)} \frac{(1/w)}{(1/w)} = \frac{t}{2(1+t/w)} = \frac{t}{2}$$

$$L_c = L + \frac{A_c}{P}$$

$$\approx L + \frac{t}{2}$$ (t << w so make this approximation for rectangular fins)

$$= 0.05 + \frac{0.001}{2} \, [m] = 0.0505 \, [m]$$

a. $\dot{Q}_{adiabatic} = \sqrt{hPkA_c}\,(T_B - T_\infty) \tanh(mL_c)$ where $m = \sqrt{\frac{hP}{kA_c}}$

$= 4.47 \, [W] \tanh(mL_c)$

↑ prev. example

$= 4.47 \, [W] \tanh(13.55 \, [m^{-1}] \; 0.0505 \, [m])$

$= 2.66 \, [W]$

b. $R_{fin} = \frac{T_B - T_\infty}{\dot{Q}_{adiabatic}} = \frac{60°C - 30°C}{2.66 \, [W]} = 11.288 \, [K/W]$

c. $\eta_{fin} = \frac{\dot{Q}}{hA_{tot}(T_B - T_\infty)} = \frac{\dot{Q}}{h(PL_c + A_c)(T_B - T_\infty)} = \frac{2.66 \, [W]}{10 \, [W/m^2K] \times 0.202 \, [m] \times 0.0505 \, [m] \times (60° - 30°C)} = 0.87$

(A_c: negligible)

d. $\varepsilon_{fin} = \frac{\dot{Q}}{hA_B(T_B - T_\infty)} = \frac{\dot{Q}}{10 \, [W/m^2K] \; 10^{-4} \, [m] \; (60° - 30°C)} = 88.51 \gg 1$

Very effective